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# Investigation Of Ultimate Resisting Moment Of Reinforced Concrete Slabs Systems Using Yield Line Theory, Bs 8110 And Computer Soft Wares

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## Abstract

In this paper, different types of reinforced concrete slabs system of different support conditions have been analyzed using yield line theory to determine the maximum resisting moment, the result of resisting moments obtained, were compared with others those obtained by using software program (STAAD-PRO, PROKON) and BS8110. The comparison revealed different conforming by percentage range by 0.097 % and 17.81%. However, the results of this paper were clearly showed significant results became within the range of specifications can be achieved with analysis of different supports of reinforced concrete slabs by using yield line theory, BS 8110 and computer soft wares.

***Key Words:*** *yield line theory, reinforced concrete slabs, ultimate resisting moments and computer soft wares*

## 1. INTRODUCTION

The term "yield line" literally meaning is line of rupture was coined in 1921 by Ingerslev [1] to describe lines in the slab along which the bending moment is constant. In 1931 K W Johansen [2] gave the concept a geometrical meaning as lines of relative rotation of rigid slab parts.

In 1938 Gvozdev [3] had already formulated the limit analysis theorems, but his work was not widely known in the West until it was translated to English in 1960. Whereas the Pager school of plasticity was mainly concerned with metallic structures, Gvozdev's point of departure was reinforced concrete, in particular slabs, [4].

Yield line analysis was adopted by the Danish concrete code [5], and introduced into the curriculum at the Technical University of Denmark. There is anecdotic evidence to the effect that the success of Danish engineers worldwide in the decades immediately following the Second World War owed no small part to their mastery of yield line analysis, allowing them to produce efficient designs of reinforced concrete slabs of any shape and loading, [5].

In the 1960s yield line theory was the subject of considerable interest in the UK, as evidenced by a flurry of papers and monographs, including a special publication issued by Magazine of Concrete Research [6], including contributions by L L Jones, K O Kemp, C T Morley, M P Nielsen and R H Wood. A particular subject under debate was whether Johansen's yield criterion was compatible with limit analysis. Jones & Wood went so far as to state in 1967 [7] that such a criterion is useless within the strict framework of limit analysis, which must develop its own idealized criteria of yield.
In 1970, however, Braestrup [9] showed that not only is the Johansen criterion consistent with limit analysis, as evidenced by the work of Nielsen, it is indeed the only possible yield condition for a slab that allows complete solutions (coinciding upper and lower bounds) to be derived by yield line analysis. The message was brought home in 1974 when Fox [10] determined the exact yield load for the clamped, isotropic slab under uniform loading. This fairly simple case had long defied attempts of solution, and this fact had been cited as evidence of the incompatibility of yield line theory and limit analysis. Fox's analysis of the square, clamped slab is not a proper yield line solution, because it includes finite regions with a negative Gaussian curvature [10]. However, yield line analysis provides a close estimate, and by successively refining the yield line pattern, the exact solution can be approximated to any desired degree, which is the point. Slabs or plates obeying other yield conditions (egTresca or v. Mises) can also be analyzed by yield lines, but except for trivial cases the resulting upper bound will never approach the exact solution, however detailed the yield line pattern. It is interesting to note that, unbeknownst to most participants in the debate 40 years ago, limit analysis and yield line theory had for many years peacefully coexisted in the Soviet Union.

In 1995 Gerg.E.Mertz mentions that: Yield line theory offers a simplified nonlinear analytical method that can determine the ultimate bending capacity of flat reinforced concrete plates subject to distributed and concentrated loads. Alternately, yield line theory, combined with hinge rotation limits can determine the energy absorption capacity of plates subject to impulsive and impact loads. This method is especially useful in evaluating existing structures that cannot be qualified using conservative simplifying analytical assumptions. Typical components analyzed by yield line theory are basements, floor and roof slabs subject to vertical loads along with walls subject to out of plane wall loads.

One practical limitation of yield line theory is that it is computationally difficult to evaluate some mechanisms. This problem is aggravated by the complex geometry and reinforcing layouts commonly found in practice. A yield line evaluation methodology is proposed to solve computationally tedious yield line mechanisms. This methodology is implemented in a small, PC based computer program, which allows the engineer to quickly evaluate multiple yield line mechanisms [11].

GregE.Mertz obtains, Yield line theory is capable of determining the ultimate bending capacity of complex slabs, and when combined with rotation limits, yield line theory can also be used to evaluate slabs for impact loads. [11]
In 2003 Tim Gudman-Hoyer papers treats the subject Yield line Theory for Concrete Slabs Subjected to Axial Force. In order to calculate the load-carrying capacity from an upper bound solution the dissipation has to be known.
For a slab without axial force the usual way of calculating this dissipation is by using the normality condition of the theory of plasticity together with the yield condition. This method is equivalent to the original proposal by K. W. Johansen. This method has shown good agreement with experiments and has won general acceptance.
The dissipation in a yield line is calculated on the basis of the Coulomb yield condition for concrete in order to verify K. W. Johansen's method. It is found that the calculations lead to the same results if the axes of rotation are the same for adjacent slab parts. However, this is only true if the slab is isotropic and not subjected to axial load.
An evaluation of the error made using K. W. Johansen's proposal for orthotropic rectangular slabs is made and it is found that the method is sufficiently correct for practical purposes.

For deflected slabs it is known that the load-carrying capacity is higher. If it is assumed that the axis of rotation corresponds to the neutral axis of a slab part and the dissipation is found from the moment capacities about these axes K. W. Johansen's proposal may be used to find the load- carrying capacity in these cases too. In this paper this is verified by comparing the results with numerical calculations of the dissipation. Also for deflected slabs it is found that the simplified method is sufficiently correct for practical purposes.

The same assumptions are also used for rectangular slabs loaded with axial force in both one and two directions and sufficiently good agreement is found by comparing the methods. Interaction diagrams between the axial load and the transverse load are developed at the end of the paper for both methods. Different approaches are discussed.

Only a few comparisons between experiments and theory are made. These indicate that the theory may be used if a proper effectiveness factor is introduced and the deflection at failure is known.
If the deflection is unknown an estimate of the deflection based on the yield strains of the concrete and the reinforcement seems to lead to acceptable results [12].
In this research an analysis of reinforced concrete slab was done by applying yield line method with depending on virtual work method.

## 2. OBJECTIVES OF THE RESEARCH

The objectives of this study are:

1- To apply the yield line theory to obtain the ultimate resistance moments of different types of slabs.
2- To compare the results obtained by yield line theory with that obtained by software program STAAD-PRO, PROKON, and BS8110.

## 3. METHODOLOGY

### 3.1 Method of Solution by Using Yield Line Theory

Once a failure pattern has been postulated two methods of solution are available in order to find the relation between the ultimate resistance moments in the slab and the ultimate load. Since the moment and the load are equilibrium when the yield line pattern has formed, the slightest increment in load will cause the structure to deflect. When this increase in load is infinitesimal, the work done on the slab while the yield lines are rotating must be equal to the loss of work due to the load deflecting.

Thus, if a point on the slab is given a virtual deflection take place along the yield lines. The internal work done on the slab will be the sum of the rotations in the yield lines multiplied by the resisting ultimate moments, while the external loss of work will be the sum of the loads multiplied by their respective deflections. When the internal and external work is equated, we have the relations between the ultimate resistance moments in the slab and the ultimate load will be obtained.

### 3.2 The 10% rule

A 10% margin on the design moments should be added when using the virtual work method or formulae for two-way slabs to allow for the method being upper bound and to allow for the effects of corner levers [13]. The addition of 10% to the design moment in two-way slabs provides some leeway where inexact yield line solutions have been used and some reassurance against the effects of ignoring corner levers. At the relatively low stress levels in slabs, a 10% increase in moment equates to a 10% increase in the reinforcement design.

The designer may of course chase in search of a more exact solution but most pragmatists are satisfied to know that by applying the 10% rule to a simple analysis their design will be on the safe side without being unduly conservative or uneconomic. The 10% rule can and usually is applied in other circumstances where the designer wants to apply engineering judgment and err on the side of caution. The only situations where allowances under this '10% rule' may be inadequate relate to slabs with acute corners and certain configuration of slabs with substantial point loads or line loads. In these cases guidance should be sought from specialist literature.

### 3.3 Serviceability and Deflections

Yield Line Theory concerns itself only with the ultimate limit state. The designer must ensure that relevant serviceability requirements, particularly the limit state of deflection, are satisfied. Deflection of slabs should be considered on the basis of elastic design. This may call for separate analysis but, more usually, deflection may be checked by using span/effective depth ratios with ultimate (i.e. yield line) moments as the basis. Such checks will be adequate in the vast majority of cases.

### 3.4 Analysis and Design of R.C. Slabs Using BS8110

Any design process is governed by the recommendations of a specific code of practice. In the UK, BS 8110 clause 3.5.2.1 says Alternatively Johansen's Yield Line method may be used for solid slabs. The proviso is that to provide against serviceability requirements, the ratio of support and span moments should be similar to those obtained by elastic theory. This sub-clause is referred to in clauses 3.6.2 and 3.7.1.2 making the approach also acceptable for ribbed slabs and flat slabs.

### 3.5 Software Program

General purpose software suite for structural engineers involved in analysis and design of structures, STAAD-Pro and PROKEN. The structural analysis and design software, **STAAD-*Pro*** was developed for practicing engineers. For static, pushover, dynamic, P-delta, buckling or cable analysis, STAAD-*Pro* is the industry standard. PROKON provides engineers with tools to streamline their Workflow in the structural and geotechnical spheres. The tools are modular, but all are launched from the Prokon Calcpad.

## 4. DISCUSSION OF THE RESULTS

### 4.1 Discussion of Flat Slab System

The Fig 1 shown below is the plan of flat slab has three equal spans at direction X of 6 m length and three spans at direction Y of length 6 m for the edges spans and 4 m length for middle span. The slab is subjected to uniformly distributed load of 20 kN/m$^2$. By considering a reasonable pattern of positive and negative yield lines is that shown in Fig 1 and with following the procedure explained at previous Chapter, the ultimate resisting moment (MP) can be obtained for each panel as named in Fig 1.

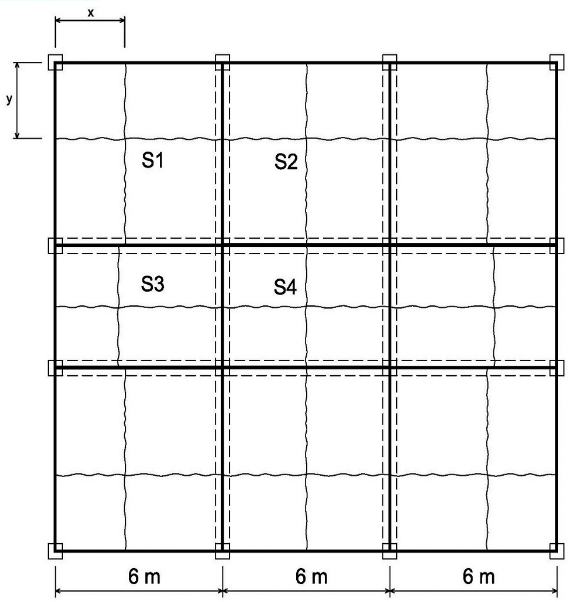


**Fig - 1**: Plan of Flat Slab System with Expected of Yield Line Pattern

#### 4.1.1 Analysis of External Corner Panel S1

Panel S1 is the square panel has length of 6 m each with two adjacent edges discontinuous and continuous in other tow edges, by considering a reasonable pattern of positive and negative yield lines is that shown in Fig 2, we will determine the Mp using work method.

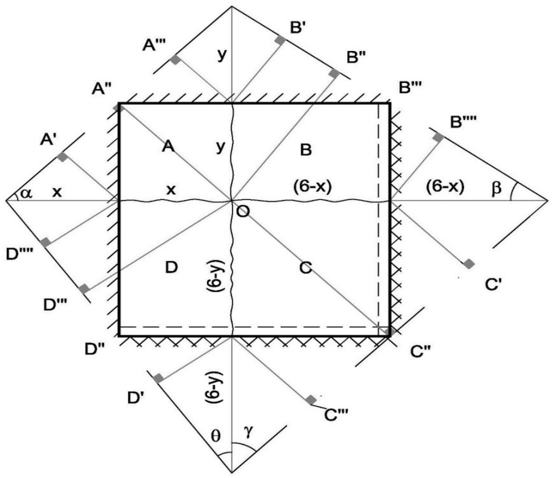


**Fig - 2**: Expected yield line Pattern of External Corner Panel S1

### 4.1.2 Analysis of Edge Panel S2

Panel S2 is the square panel has length of 6m with one edge discontinuous and three edges continuous, by considering a reasonable pattern of positive and negative yield lines is that shown in Fig 3, we will determine the $M_{max}$ using work method.

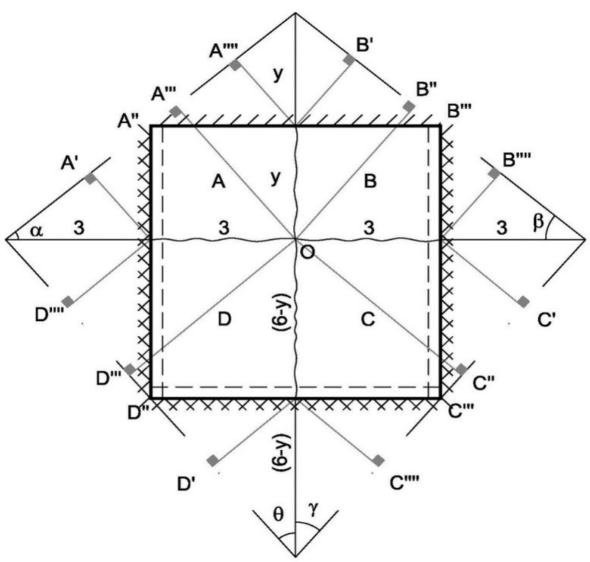


**Fig - 3**: Expected yield line Pattern of the edge Panel S2

### 4.1.3 Analysis of edge Panel S3

Panel S3 is the rectangular panel has length of 6m and 4m width with one edge discontinuous and three edges continuous, by considering a reasonable pattern of positive and negative yield lines is that shown in Fig 4, we will determine the Mp using work method.

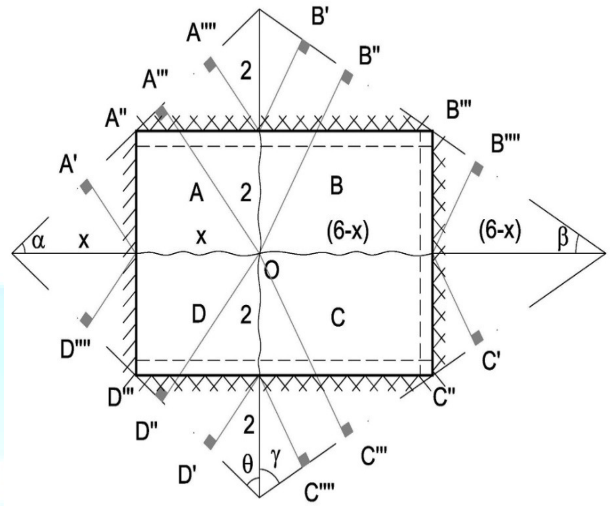


**Fig - 4**: Expected yield line Pattern of the edge Panel S3

### 4.1.4 Analysis of edge Panel S4

Panel S4 is the rectangular panel has length of 6m and width 4m with four edges continuous, by considering a reasonable pattern of positive and negative yield lines is that shown in **Fig 5**, we will determine the $M_p$ using virtual work method.

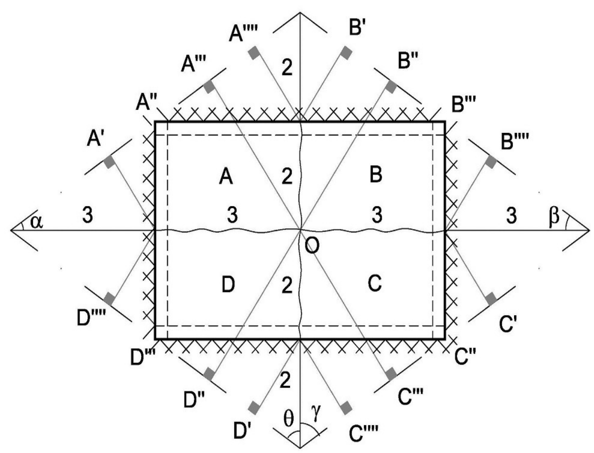


**Fig - 5**: Expected yield line Pattern for the interior Panel S4

The results obtained for the ultimate resisting moments for each panel of reinforced concrete flat slabs were summarized at Table 1 and were compared with value obtained by using STAAD-Pro Software.

**Table -1:** The Ultimate Resisting Moments for Flat Slab

| Panel | Value of (MP) by yield line theory kN.m/m | Value of (MP) by STAAD-pro kN.m/m | Difference % |
|---|---|---|---|
| S1 | 61.80 | 61.74 | 0.097% |
| S2 | 52.00 | 52.96 | -1.85% |
| S3 | 48.02 | 47.22 | 1.67% |
| S4 | 27.6 | 26.09 | 5.47% |

From results of ultimate resisting moments for R.C. flat slab system it was appeared that the difference about 0.097% to 5.47% in comparison of with yield line theory.

### 4.2 Analysis of Beam Slab System

The Fig 6 below is the plan of slab with beam has three equal spans at X direction of 6 m length and three spans at Y direction of length 6 m for the edges spans and 4 m length for middle span. , The slab is subjected to uniformly distributed load of 20 kN/m$^2$. By considering a reasonable pattern of positive and negative yield lines is that shown in Fig 6, and with following the procedure explained at previous Chapter, the ultimate moment (MP) can be obtained for each panel as named in Fig 6.

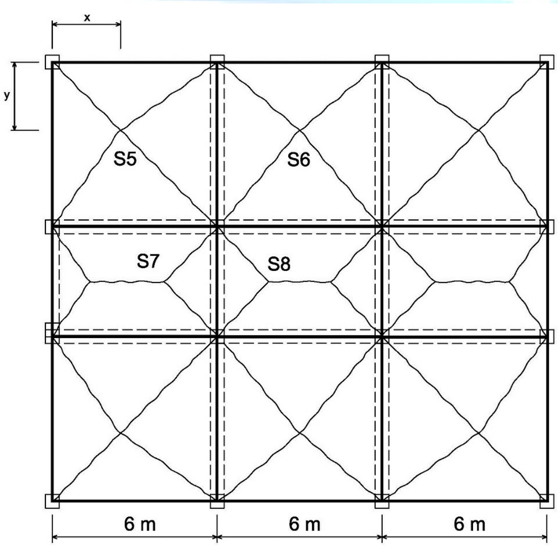


**Fig - 6**: Plan of beam Slab System with Expected of Yield Line Pattern

#### 4.2.1 Analysis of External Corner Slab S5

Panel S5 is the square panel has length of 6 m each with two adjacent edges discontinuous and continuous in other two sides, by considering a reasonable pattern of positive and negative yield lines is that shown in Fig 7, we will determine the MP using work method.

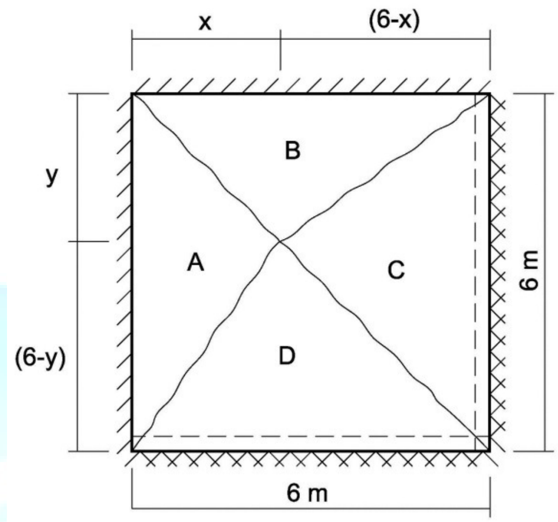


**Fig - 7**: Expected Yield line Pattern of External Corner Panel S5

#### 4.2.2 Analysis of Edge Slab S6

Panel S6 is the square panel has length of 6m with one edge discontinuous and three edges continuous, by considering a reasonable pattern of positive and negative yield lines is that shown in Fig 8, we will determine the Mp using work method.

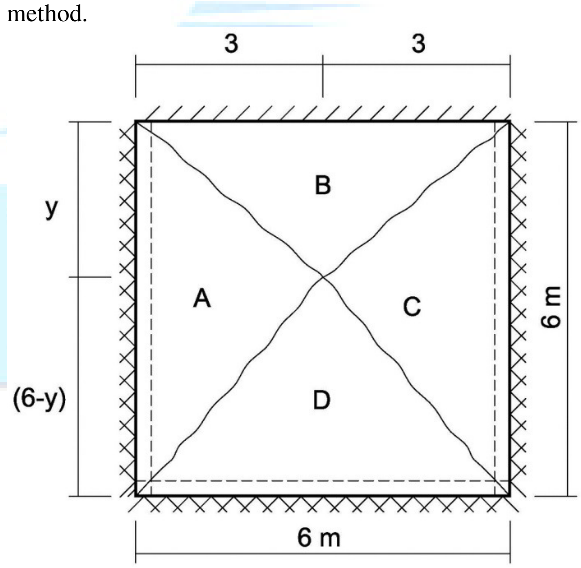


**Fig - 8**: Expected Yield Line Pattern of edge Panel S6

### 4.2.3 Analysis of Edge Slab S7

Panel S7 is the rectangular panel has length of 6m and width 4m with one edge discontinuous and three edges continuous, by considering a reasonable pattern of positive and negative yield lines is that shown in **Fig 9**, we will determine the $M_p$ using work method.

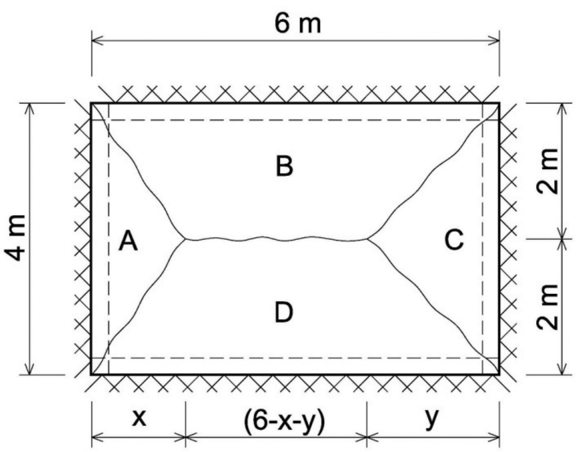


**Fig - 9**: Expected yield line Pattern of edge Panel S7

### 4.2.4 Analysis of Interior Slab S8

Panel S8 is the rectangular panel has length of 6m and width 4m with four edges continuous, by considering a reasonable pattern of positive and negative yield lines is that shown in **Fig 10**, we will determine the $M_p$ using work method.

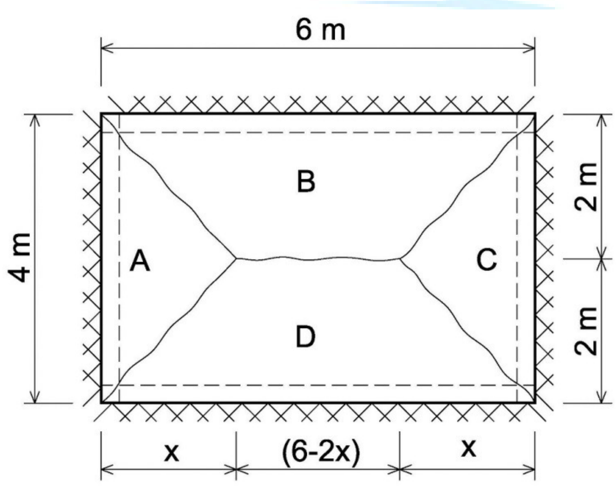


**Fig - 10**: Expected Yield Line Pattern of Interior Panel S8

The results obtained for the ultimate moments for each panel of reinforced concrete slabs with beams are summarized as shown in Table 2 and compared with others obtained by using STAAD-Pro Software, and BS8110.

**Table - 2:** The Ultimate Resisting Moments for Beam slab System

| Panel | Value of ($M_p$) by yield line theory KN.m/m | Value of ($M_p$) by BS 8110 KN.m/m | Difference% | Value of ($M_p$) by STAAD-pro KN.m/m | Difference% |
|---|---|---|---|---|---|
| S5 | 22.65 | 24.48 | 8.08% | 25.8 | 13.91% |
| S6 | 19.14 | 20.88 | 9.09% | 20.02 | 4.6% |
| S7 | 13.86 | 13.76 | 0.72% | 14.95 | 7.86% |
| S8 | 12.85 | 12.8 | 0.39% | 12.76 | 0.70% |

The results of ultimate resisting moments obtained were shown in Table 2 for the R.C. beam slab system that were compared with others those obtained by using STAAD-Pro software and BS8110. The comparison revealed different conforming by percentages range by 0.70 % to 13.91% when comparing with STAAD-PRO and range by 0.39 % to 9.09 % when comparing to BS8110.

### 4.3 Special Beam Slab System of Different Support conditions

### 4.3.1 Analysis of corner square slab S9

Panel S9 square slab length of 5m with edges continuous in two sides and simply supported in other two sides, by considering a reasonable pattern of positive and negative yield lines is that shown in **Fig 11**, the ultimate resisting moment $M_p$ can be determine using virtual work method.

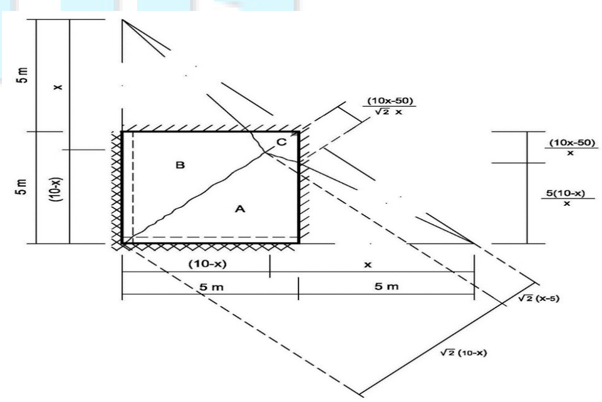


**Fig - 11**: Expected yield line Pattern for the Corner Panel S9

### 4.3.2 Analysis of rectangular slab S10

A rectangular slab of length 6m and width 4m with continuance edge in one side and simply supported in two sides, and free edge in one, by considering a reasonable pattern of positive and negative yield lines is that shown in **Fig 12**, we will determine the $M_{max}$ using work method.

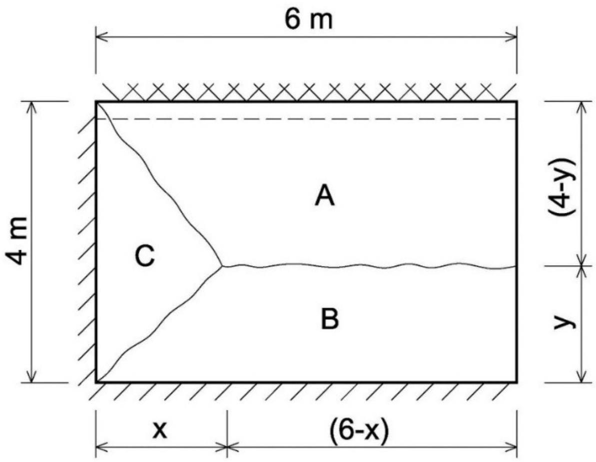


**Fig - 12**: Expected yield line Pattern of Corner Panel S10

The results obtained for the ultimate moments for the panel S10 are summarized at Table 3 and compared with others obtained by using Prokon Software, and BS8110.

**Table - 3:** The Ultimate Resisting Moments for Panels S10

| panel | Value of ($M_u$) by yield line theory kN.m/m | Value of ($M_u$) by BS8110 Table 3.14 kN.m/m | Different % | Value of ($M_u$) by Prokon kN.m/m | Different % |
|---|---|---|---|---|---|
| S9 | 32.00 | 31.00 | 3.125% | 37.7 | 17.% |
| S10 | 22.00 | 20.16 | 8.36% | 22.9 | 4.09% |

The results of ultimate resisting moments obtained from Table 3 for the R.C. slabs that have special condition of supports that were compared with others those obtained by using PROKON software and BS8110. The comparison revealed different conforming by percentages range by 4.09 % to 17.81% when comparing with PROKON and range by 3.125 % to 8.36 % when comparing with BS8110.

## 5. CONCLUSIONS

On Basis of this study, Conclusions that can be drawn are as follows:

1. By using yield line theory, different types of reinforced concrete slabs are used to determine the ultimate resisting moments and their locations.
2. One of the most popular methods of application in yield line theory is the virtual work method that was used in this research to analysis and assessment different models of reinforced concrete slabs (beam slab and flat slab system) of different shapes and different support conditions, In addition to slabs that have special condition of supports.
3. The percentages range between 0.097 % to 5.47 % of the results of bending moments for the reinforced concrete flat slab system that were compared with others those obtained by using STAAD-Pro software. This results confirm to the software program STAAD-Pro with manual calculations, the results were classified as very good once.
4. The percentages range between 0.70 % and 13.91% of the results of bending moments for the reinforced concrete beam slab system that were compared with others those obtained by using STAAD-Pro software. These results confirm to the software program STAAD-Pro with hand calculations, the results were classified as good once. The same results from manual calculations were compared with others those obtained by using BS8110 the percentages range of difference between 0.39 %and 9.09 %. This results obtained by BS8110 and manual calculations were closer than obtained by STAAD-Pro, the results were classified good.
5. The percentages range between 4.09 % and 17.81% of the results of bending moments for the reinforced concrete slabs that have special condition of supports were compared with others those obtained by using PROKON software. These results related to the software program PROKON with hand calculations, the results were classified very good results. The same results from hand calculations were compared with others those obtained by using BS8110 the percentages range of difference between 3.125 %and 8.36 %. These results confirm to the BS8110 with hand calculations and it was closer than results from PROKON, the results were classified very good results.
6. As the general, the results of this study were clearly demonstrated that acceptable and close results can be achieved with analysis by using yield line theory.

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